

Commentary on the Habilitation thesis

**CATEGORICAL STRUCTURES FOR HIGHER
DIMENSIONAL UNIVERSAL ALGEBRA**

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This habilitation thesis presents recent developments in categorical algebra, motivated by higher dimensional structures. It makes significant advances to two-dimensional universal algebra, algebraic weak factorisation systems, skew monoidal categories, monads and their associated theories and to homotopically enriched category theory.

The thesis comprises six chapters. The first chapter gives an overview of the main topics covered in the thesis and presents our main contributions, as well as providing the broader context in which these contributions were made. These contributions are collected from five recent papers [1, 3, 2, 4, 5], contained in Chapter 2 to 6. Chapter 2 provides an extension of Beck's monadicity theorem from classical universal algebra to two-dimensional universal algebra, dealing with pseudo, lax and colax morphisms. Chapter 3 makes significant developments to the theory of algebraic weak factorisation systems, introducing a new more powerful notion of cofibrant generation and providing a simple new recognition theorem that makes these structures much easier to handle and recognise. Chapter 4 builds a new bridge between skew monoidal categories on the one hand and homotopy theory and 2-category theory on the other. Chapter 5 introduces new classes of monads and theories, motivated by variants of the nerve theorem appropriate to higher dimensional categories. Chapter 6 introduces and studies homotopically enriched category theory of a flavour adapted to capture higher categories with algebraic structure, as suggested by the term *higher dimensional universal algebra* in the thesis title.

These results are based on my two single-author papers [1, 3], two joint papers with Richard Garner [2, 4] and one joint paper [5] with Steve Lack and Lukáš Vokřínek. The first four have all been published in high ranking journals whilst the fifth [5] is a recent 2020 preprint, currently submitted for publication. The three joint papers are the results of continuous back and forth collaboration and, accordingly, the contributions of all collaborators are to be regarded as equivalent. In this regard, my contribution to the two papers with Garner is 50% and to the three-author paper is 33%.

REFERENCES

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